WebCT Truth Table Assignments:
An Example

This is an example of how to do a "**truth table choice" problem**, of the sort that you will have on WebCT quizzes and exams.

Problem: For the following argument, (i) **list each sentence** that shows up in the truth table for the argument; then (ii) **state**, **for each sentence** you've listed, **which** of the truth table choices **is the truth table for that sentence**; and finally (iii) state whether the argument is **valid** or **invalid**.

Argument:

1. $\sim(P \wedge Q)$
2. P

$\therefore\ \sim Q$

Truth Tables Choices:

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
0	1	0	1	0	0	0	1
0	0	1	0	1	1	0	1
1	0	0	1	1	0	1	0
1	0	0	0	1	1	0	0

Discussion: To complete this problem, we must first build a truth for each sentence in the argument (the two premises, and the conclusion).

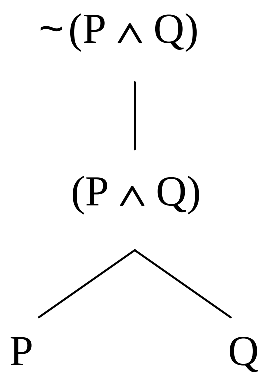
All the sentences are built up from sentence letters "P" and "Q," so our truth table for this argument will begin with these sentence letters.

P	Q

We build truth tables for these sentence letters in the usual manner.

P	**Q**
1	1
1	0
0	1
0	0

The first premise, "~(P ∧ Q)," was built up like so.



The truth table for Premise (1) will follow these steps. "P" and "Q" are already in the truth table; so to finish the truth table for this premise, we just need to add the remaining two steps – "(P ∧ Q)" and "~(P ∧ Q)".

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**
1	1		
1	0		
0	1		
0	0		

The second premise is just "P," which is already in the truth table.

The conclusion, "~Q," is not yet in the truth table; so we add this sentence.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1			
1	0			
0	1			
0	0			

"(P ∧ Q)" is a conjunction, following the Conjunction Rule: a conjunction is only truth when both its parts are true.

●	▲	(● ∧ ▲)
1	1	1
1	0	0
0	1	0
0	0	0

So "(P ∧ Q)" is true in the first valuation.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	**1**		
1	0			
0	1			
0	0			

It's false in Valuations (2) through (4).

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1		
1	0	**0**		
0	1	**0**		
0	0	**0**		

"~(P ∧ Q)" is the negation of "(P ∧ Q)," and follows the Negation Rule.

●	~●
1	0
0	1

So when the original sentence, "(P ∧ Q)," is true (Valuation 1), its negation is false.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1	**0**	
1	0	0		
0	1	0		
0	0	0		

When "(P ∧ Q)"is false (Valuations 2 through 4), its negation is true.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1	0	
1	0	0	**1**	
0	1	0	**1**	
0	0	0	**1**	

"~Q" also follows the Negation Rule: when "Q" is true (Valuations 1 and 3), "~Q" is false.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	**1**	1	0	**0**
1	0	0	1	
0	**1**	0	1	**0**
0	0	0	1	

When “Q” is false (Valuations 2 and 4), “~Q” is true.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1	0	0
1	**0**	0	1	**1**
0	1	0	1	0
0	**0**	0	1	**1**

With the truth table for the argument completed, we are in a position to complete the problem. We needed to ***first*** make a list of each of the sentences appearing in the truth table. That’s easy: these are just the sentences listed across the top of the truth table. So our list is as follows.

Sentences:

P
Q
(P ∧ Q)
~(P ∧ Q)
~Q

Second, we needed to state, for each of these sentences, which of the numbered “truth table choices” is the *right* truth table for that sentence.

Truth Table Choices:

(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**
0	1	0	1	0	0	0	1
0	0	1	0	1	1	0	1
1	0	0	1	1	0	1	0
1	0	0	0	1	1	0	0

Checking with our completed truth table for this argument, we see that "**P**" takes truth table choice **(viii)**; "**Q**" takes **(iv)**; "**(P ∧ Q)**" takes **(ii)**; "**~(P ∧ Q)**" takes **(v)**; and "**~Q**" takes **(vi)**.

So we put the matching truth table number next to each sentence in our list.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1	0	0
1	0	0	1	1
0	1	0	1	0
0	0	0	1	1

Truth Table Choices:

(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**
0	**1**	0	**1**	**0**	**0**	0	**1**
0	**0**	1	**0**	**1**	**1**	0	**1**
1	**0**	0	**1**	**1**	**0**	1	**0**
1	**0**	0	**0**	**1**	**1**	0	**0**

Sentences, and Matching Truth Tables:

P: (viii)
Q: (iv)
(P ∧ Q): (ii)
~(P ∧ Q): (v)
~Q: (vi)

Finally, we must – based on our truth table for the argument – say whether the argument is valid or invalid. This is settled by first picking out the valuations where all of the premises are true. Only Valuation (2) makes all the premises true.

1 P	Q	(P ∧ Q)	**2** ~(P ∧ Q)	**∴** ~Q
1	1	1	0	0
1	0	0	**1**	1
0	1	0	1	0
0	0	0	1	1

And in that valuation, the conclusion is also true.

1 P	Q	(P ∧ Q)	**2** ~(P ∧ Q)	**∴** ~Q
1	1	1	0	0
1	0	0	**1**	**1**
0	1	0	1	0
0	0	0	1	1

Since whenever the premises are (all) true, the conclusion is also true, this argument is **valid**. Putting in our verdict on the argument's validity completes the assignment.

Final Answer:

Sentences, and Matching Truth Tables:

P: (viii)
Q: (iv)
(P ∧ Q): (ii)
~(P ∧ Q): (v)
~Q: (vi)

Verdict: Argument is Valid

This is how we complete truth table problems in a WebCT quiz or exam. As this example illustrates, you ***will*** need to do the truth table for the argument (on paper) in order to answer the questions properly. But by doing the problem in this "truth table choice" fashion, we avoid trying to type out truth tables in the text boxes of WebCT – which would be quite difficult and messy.